The Realities of Solar Power and Solar Energy for the Real World

A simple Navigational Problem

Imagine that you are at a particular locale at a given time. Furthermore, imagine that you are on on a journey to another locale and that you are navigating by the stars. Now imagine that you are carrying an astronomical almanac, a good clock synchronized to the time system used by the astronomical almanac, a compass, and a simple sextant.

At the aforementioned given time a certain star will stand at zenith directly over the destination locale. From you locale you use the compass to determine the compass azimuth angle of the star and the sextant to determine the altitude of the star above the level horizon. Then you apply the basic formulas from spherical trigonometry to calculate the distance to the destination locale as well as your current latitude and displacement of longitude. You already know the latitude and the longitude of the destination locale.

Your journey will be a long one. It will take many weeks to accomplish. You cannot follow a simple course. Geography and political divisions will require adjustments. The course must be repeatedly recalculated. On some nights the sky will be clouded over or you will be in the midst of an obscuring forest. You must accept these things as normal.

Here are the variables that you will be using in the navigational calculations. This is assuming that the star is directly above the destination locale and that the Terrestrial latitude of the destination locale is known.

1. [a]: This variable represents the known altitude of the star with respect to a point directly above your current position.
2. [b]: This variable represents the unknown latitude of your current position with respect to the Terrestrial North Pole.

3. [c]: This variable represents the known latitude of the destination locale with respect to the Terrestrial North Pole.
4. [d]: This variable represents the unknown distance between the current locale and the destination locale. This calculation requires that [a] be expressed as radians.
5. [α]: This variable represents the unknown displacement of longitude between the current locale and the destination locale.
6. [γ]: This variable represents the known azimuth angle of the star from the current locale.
7. [alt]: This variable represents the known altitude of the star with respect to the level horizon.
8. [lat1]: This variable represents the unknown latitude of the current locale with respect to Terrestrial Equator.
9. [lat2]: This variable represents the known latitude of the destination locale with respect to the Terrestrial Equator.

There is an additional known variable [r] representing the radius of the planet (i.e. Earth).

Given [r], [lat2] [alt], and [γ] --- [alt] and [lat2] are independent.

r = 6,371 km (radius of Earth)

$$a = \frac{\pi}{2} - alt \qquad c = \frac{\pi}{2} - lat2 \qquad d = a \cdot r \qquad alt > 0 \qquad alt > lat2$$

$$\cos(c) = \cos(a)\cdot\cos(b) + \sin(a)\cdot\sin(b)\cdot\cos(\gamma)$$

$$x = \cos(a)^2 + \sin(a)^2\cdot\cos(\gamma)^2 \qquad \cos(b) = \frac{y - \sqrt{y^2 - 4\cdot x\cdot z}}{2\cdot x} \qquad \gamma > \frac{\pi}{2}$$

$$y = 2\cdot\cos(a)\cdot\cos(c)$$

$$z = \cos(c)^2 - \sin(a)^2\cdot\cos(\gamma)^2 \qquad \cos(b) = \frac{y + \sqrt{y^2 - 4\cdot x\cdot z}}{2\cdot x} \qquad \gamma < \frac{\pi}{2}$$

$$b = \operatorname{acos}(\cos(b)) \qquad \alpha = \operatorname{acos}\left(\frac{\cos(a) - \cos(b)\cdot\cos(c)}{\sin(b)\cdot\sin(c)}\right)$$

Times and Azimuths of Sunrise and Sunset

It is fairly straight-forward to calculate the times and the azimuths of sunrise and sunset for a level unobstructed plain. However, we live in a shared world that is neither level nor unobstructed. Furthermore, we live under an atmosphere equivalent to a minimum of about 30 feet of clear water through which the rays of the Sun must pass. The volume of the atmosphere significantly increases when the Sun is near the horizon. There are trees, hills, mountains, and man made structures to contend with.

We must not seek 100% of our wants. We must be willing to compromise and share. For this reason we must apply an obstruction altitude that the Sun must clear before we can use its direct rays.

Here are the variables to be applied. They are the basic variables (a, b, c, α, β, γ) from spherical trigonometry. An [*o] representing “obstruction” will be used as a suffix for all but [lat], [dec], [b], and [c] which are the universally constant givens in all the equations. The times in radians will be with respect to the Sun at zenith. The azimuths in radians will be with respect to the Terrestrial North Pole.

- [alto]: This variable represents the minimum altitude that the Sun must clear to be visible with respect to the level horizon.
- [lat]: This variable represents the Terrestrial latitude of the observer with respect to the Terrestrial Equator. North is positive (+) and South is negative (-).
- [dec]: This variable represents the declination of the Sun with respect to the Celestial Equator. North is positive (+) and South is negative (-).
- [ao]: This variable represents the minimum altitude that the Sun must clear to be visible with respect to a point directly above the observer.

- [b]: This variable represents the Terrestrial latitude of the observer with respect to the Terrestrial North Pole
- [c]: This variable represents the declination of the Sun with respect to the Celestial North Pole.
- [αo]: This variable represents the paired times of the "obstructed" sunrise and sunset with respect to the zenith of the Sun.
- [γo]: This variable represents the paired azimuths of the "obstructed" sunrise and sunset with respect to the Terrestrial North Pole.

In this illustration the three given angles have already been converted from the original degrees to radians. The lower portion shows the conversion to the conventional forms.

$$ao = \frac{\pi}{2} - alto \quad b = \frac{\pi}{2} - lat \quad c = \frac{\pi}{2} - dec$$

$$\alpha o = +/- \ acos\left(\frac{\cos(ao) - \cos(b) \cdot \cos(c)}{\sin(b) \cdot \sin(c)}\right) \quad -1 < \cos(\alpha 0) < 1$$

$$\gamma o = +/- \ acos\left(\frac{\cos(c) - \cos(ao) \cdot \cos(b)}{\sin(ao) \cdot \sin(b)}\right) \quad -1 < \cos(\gamma 0) < 1$$

$$\text{Time of sunrise in hours from nadir} = 12 - \alpha o \cdot \frac{12}{\pi}$$

$$\text{Azimuth of sunrise in degrees from north} = \gamma o \cdot \frac{180}{\pi}$$

$$\text{Time of sunset in hours from nadir} = 12 + \alpha o \cdot \frac{12}{\pi}$$

$$\text{Azimuth of sunset in degrees from north} = 360 - \gamma o \cdot \frac{180}{\pi}$$

The calculated times of sunrise and sunset for the "obstructed" altitude will be applied in the power and energy equations.

Simple Adjustments to Solar Power and Solar Energy for the Obstructions of the Real World

Here is the bottom line. You cannot receive solar power nor accumulate solar energy from direct sunlight if the direct sunlight is obscured. This next discussion is still for a level plain. The suffix [*o] indicates that the variable pertains to the obstructed horizon. Here is a list of the applicable variables:

- [alto]: This variable represents the minimum altitude that the Sun must clear to be visible with respect to the level horizon.
- [lat]: This variable represents the Terrestrial latitude of the observer with respect to the Terrestrial Equator. North is positive (+) and South is negative (-).
- [dec]: This variable represents the declination of the Sun with respect to the Celestial Equator. North is positive (+) and South is negative (-).
- [ao]: This variable represents the minimum altitude that the Sun must clear to be visible with respect to a point directly above the observer.
- [b]: This variable represents the Terrestrial latitude of the observer with respect to the Terrestrial North Pole
- [c]: This variable represents the declination of the Sun with respect to the Celestial North Pole.
- [αo]: This variable represents the paired times of the “obstructed” sunrise and sunset with respect to the zenith of the Sun.
- [p]: This variable is a convenient holder for the product of cos(b) and cos(c).
- [q]: This variable is a convenient holder for the product of sin(b) and sin(c).

- [sol]: This variable represents the solar constant. By convention it is expressed as kilowatts per square meter. The solar constant varies throughout the year.
- [pow]: This variable represents the power available from solar radiation on a level surface. It is by convention expressed as kilowatts per square meter.
- [cono]: This variable represents the accumulated solar energy from the obstructed sunrise to zenith or as the case may be, the accumulated solar energy from zenith to the obstructed sunset. The total accumulated solar energy is twice the value of cono. It is used as a constant for the solar energy integral to measure the accumulated solar energy from the obstructed sunrise. It is expressed as kilowatt-hours per square meter.
- [eneo]: This variable represents the accumulated solar energy beginning at the obstructed sunrise and ending at the obstructed sunset. It is expressed as kilowatt-hours per square meter.

$$ao = \frac{\pi}{2} - alto \quad a = \frac{\pi}{2} - alt \quad b = \frac{\pi}{2} - lat \quad c = \frac{\pi}{2} - dec$$

$$p = \cos(b)\cdot\cos(c) \qquad q = \sin(b)\cdot\sin(c)$$

$$\cos(\alpha o) = \frac{\cos(ao) - p}{q} \qquad \alpha o = +/-\ \mathrm{acos}\left(\frac{\cos(ao) - p}{q}\right)$$

$$pow = sol\cdot\cos(a) = sol\cdot(p + q\cdot\cos(\alpha)) \qquad |\alpha| \le |\alpha o|$$

$$pow = 0 \qquad |\alpha| > |\alpha o|$$

$$cono = sol\cdot\frac{12}{\pi}\cdot(p\cdot\alpha o + q\cdot\sin(\alpha o))$$

$$eneo = sol\cdot\frac{12}{\pi}\cdot(p\cdot\alpha + q\cdot\sin(\alpha)) + cono \qquad |\alpha| \le |\alpha o|$$

For [cono] and [eneo] angles [α] and [αo] MUST be expressed as radians. The total energy accumulation is [2 x cono].

Positional Displacement of Latitude and Longitude

The academic world is a featureless level plain. If the real world were a featureless level plain, then the featureless level plain would be covered by over a mile of ocean. In the latter case the aforementioned worldwide ocean would have great waves undulating upon its surface and would definitely not be a featureless level plain.

The real world is comprised of gentle slopes, extreme slopes, hills and mountains. However, for any slope anywhere on the Earth, there is another locale on the Earth where by projection the aforementioned slope is a level plain. This level plain is a parallel plane to the plane of the aforementioned slope. This is calculated as the positional displacement of latitude and longitude.

The solution to finding this displacement of longitude was represented in the “Simple Navigational Problem” that was used as an introduction to this document. The basic solar power and solar energy calculations are done for the displacement of the latitude. However, the difference in longitude is treated as a difference in time with respect to the meridian of the observer.

Here is the situation. At the locale of the observer is a slope. A ray is imagined to be perpendicular to the plane of the Slope. Another ray is imagined originating at the center of the Earth and passing upward through the slope. The angular difference between the two rays we will designate as [ax]. The observer is located at an angular distance from the Terrestrial North Pole with respect to the Center of the Earth that we will designate as [b]. The angular distance with respect to the Terrestrial North Pole and where the two rays coincide we will designate as [bx]. The slope will face a compass azimuth angle with respect to the Terrestrial North Pole which we will designate as [γx]. The angular distance between [b] and [bx] we will designate as the displaced time angle [αx]

Here are the variables for this problem. The suffix [x] indicates that the variable pertains strictly to the displacement.

- [lat]: This variable represents the latitude of the Slope with respect to the Terrestrial Equator. (N = +, S = -).
- [latx]: This variable represents the displaced latitude with respect to the Terrestrial Equator. (N = +, S = -).
- [ax]: This variable represents the angle of the slope from the horizontal, or alternately, the angle of separation of a perpendicular ray to the slope and a point directly above the slope.
- [b]: This variable represents the latitude of the slope with respect to the Terrestrial North Pole.
- [bx]: This variable represents the latitude of the displaced slope with respect to the Terrestrial North Pole.
- [αx]: This variable represents the displaced longitude which in turn represents the displacement in time with respect to the calculations.
- [γx]: This variable represents the compass azimuth that the slope is facing with respect to the Terrestrial North Pole.

$$b = \frac{\pi}{2} - lat \qquad latx = \frac{\pi}{2} - bx$$

$$\cos(bx) = \cos(ax)\cdot\cos(b) + \sin(ax)\cdot\cos(b)\cdot\cos(\gamma x)$$

$$bx = \operatorname{acos}\left(\cos(ax)\cdot\cos(b) + \sin(ax)\cdot\cos(b)\cdot\cos(\gamma x)\right)$$

$$\cos(\alpha x) = \frac{\cos(ax) - \cos(b)\cdot\cos(bx)}{\sin(b)\cdot\sin(bx)}$$

$$\alpha x = \operatorname{acos}\left(\frac{\cos(ax) - \cos(b)\cdot\cos(bx)}{\sin(b)\cdot\sin(bx)}\right) \qquad 0 \le \gamma x \le \pi$$

$$\alpha x = -\operatorname{acos}\left(\frac{\cos(ax) - \cos(b)\cdot\cos(bx)}{\sin(b)\cdot\sin(bx)}\right) \qquad \pi \le \gamma x \le 2\cdot\pi$$

Conflict and Resolution

Here we have the situation where the slope represents a displaced latitude and longitude where the slope would be a level plain. In both cases the calculations for power and energy as a function of the time of the day are with respect to their respective solar zeniths of the respective locales. Unless the “real” and the “virtual” locales reside on the same meridian, there is going to be conflict in the apparent timing of the events. Given that the “real” meridian of the observer must always take precedence, the calculations must allow for the time the Sun clears the obstructed altitude in the “real” locale.

Let us begin with a declaration and a definition of the variables to be employed. Many of the variables will be different from previous variables given for the same thing. There will be a reasonable amount of logic in the selection, but logic and reason have their limits. The first variables to be listed will be the given parameters.

1. [latitude]: This variable represents the Terrestrial latitude of the site. It is expressed as degrees with respect to the Terrestrial Equator. North latitude is assigned a positive (+) value and South latitude is assigned a negative (-) value.
2. [declination]: This variable represents the Celestial declination of the Sun. It is expressed as degrees with respect to the Celestial Equator. North Declination is assigned a positive (+) value and South Declination is assigned a negative (-) value.
3. [sc]: This variable represents the solar constant for the particular day. It is expressed as kilowatts per square meter. The mean solar constant is 1.373 kw/m^2. However, at perihelion it rises to 1.420 kw/m^2 and drops to 1.329 kw/m^2 at aphelion.

4. [obstruction]: This variable represents the necessary altitude angle of the Sun required to clear the surrounding obstacles with respect to the level horizon. It is expressed as degrees with respect to the level horizon. Above the level horizon is expressed as a positive (+) value and below the level horizon is expressed as a negative (-) value. In practice, the negative values are rarely used because solar power and solar energy require that the Sun be above the level horizon.
5. [slope]: This variable represents the positive (+) angle that a slope makes with respect to the level horizon. Alternately, it represents an angle that a ray perpendicular to the slope makes with respect to a point directly above the site.
6. [direction]: This variable represents the compass azimuth that a slope faces with respect to the Terrestrial North Pole. It is expressed as degrees with respect to the Terrestrial North Pole. North is alternately assigned a value of 0º or 360º. East, the direction of sunrise, is assigned a value of 90º. South is assigned a value of 180º. West, the direction of sunset, is assigned a value of 270º.

These are the six essential given parameters required for the solar power and solar energy equations. In mathematics radians are preferred to degrees or hours. All but the solar constant need to be converted into radians and formatted for the formulas used in spherical trigonometry.

1. [lat]: This variable represents the latitude of the site with respect to the Terrestrial Equator. It is expressed as radians with respect to the Terrestrial Equator.

2. [dec]: This variable represents the declination of the Sun with respect to the Celestial Equator. It is expressed as radians with respect to the Celestial Equator.
3. [obs]: This variable represents the aforementioned [obstruction] angle as expressed as radians
4. [dir]: This variable represents the aforementioned [direction] angle as expressed as radians.
5. [ao]: This variable represents the aforementioned [obstruction] angle with respect to a point directly overhead. It is expressed as radians,
6. [b]: This variable represents the aforementioned [latitude] with respect to the Terrestrial North Pole. It is expressed as radians.
7. [c]: This variable represents the aforementioned [declination] of the Sun with respect to the Celestial North Pole. It is expressed as radians.
8. [ax]: This variable represents the aforementioned [slope] as expressed as radians. This variable also represents the angular distance with respect to the center of the Earth between the original position and the displaced position. [ax] is the side opposite the displaced longitude [αx] to be calculated later.
9. [γx]: This variable represents the aforementioned [direction] as expressed as radians.

Here are the conversions that shall be used in this section:

$$lat = latitude \cdot \frac{\pi}{180} \qquad b = \frac{\pi}{2} - lat \qquad dec = declination \cdot \frac{\pi}{180}$$

$$obs = obstruction \cdot \frac{\pi}{180} \qquad ao = \frac{\pi}{2} - obs \qquad c = \frac{\pi}{2} - dec$$

$$ax = slope \cdot \frac{\pi}{180} \qquad \gamma x = direction \cdot \frac{\pi}{180}$$

At this point we have completed the entries of our given parameters and reduced them to a form suitable for the mathematical processing. The angles have all been reduced to radians and the solar constant has been left alone. Now we need to do the third stage which is to calculate the remainder of the constants necessary for the solar power and solar energy equations. We will first declare and define the newest variables.

1. [p]: This variable represents the product of the cosine of the site latitude with respect to the Terrestrial North Pole and the cosine of the declination of the Sun with respect to the Celestial North Pole. This is a convenience.
2. [q]: This variable represents the product of the sine of the site latitude with respect to the Terrestrial North Pole and the sine of the declination of the Sun with respect to the Celestial North Pole. This is a convenience.
3. [αo]: This variable represents the positive (+) time differential with respect to the zenith of the Sun (high noon) that the Sun clears the given obstacle.
4. [bx]: This variable represents the displaced latitude with respect to the Terrestrial North Pole. At this latitude the slope at the site would be a level plain.
5. [αx]: This variable represents the positive (+) displaced longitude with respect to the meridian of the site. At this longitude the slope at the site would be a level plain.
6. [px]: This variable represents the product of the cosine of the displaced latitude with respect to the Terrestrial North Pole and the cosine of the declination of the Sun with respect to the Celestial North Pole. This is a convenience.
7. [qx]: This variable represents the product of the sine of the displaced latitude with respect to the Terrestrial North Pole

and the sine of the declination of the Sun with respect to the Celestial North Pole. This is a convenience.

8. [αy]: This variable represents the positive (+) time differential with respect to the zenith of the Sun (high noon) that the Sun clears the level horizon of the slope with respect to the special frame of reference of the slope (i.e. the displaced latitude and the displaced longitude).

$$p = \cos(b)\cdot\cos(c) \quad q = \sin(b)\cdot\sin(c) \quad \alpha o = \operatorname{acos}\left(\frac{\cos(ao) - p}{q}\right)$$

$$bx = \operatorname{acos}(\cos(ax)\cdot\cos(b) + \sin(ax)\cdot\sin(b)\cdot\cos(\gamma x)) \quad \text{(Displaced Lat)}$$

$$\alpha x = \operatorname{acos}\left(\frac{\cos(ax) - \cos(b)\cdot\cos(bx)}{\sin(b)\cdot\sin(bx)}\right) \quad \text{(Displaced Lon)}$$

$$px = \cos(bx)\cdot\cos(c) \quad qx = \sin(bx)\cdot\sin(c) \quad \alpha y = \operatorname{acos}\left(\frac{\theta - px}{qx}\right)$$

The arc-cosines of [αo], [αx], and [αy] are all expressed as positive (+) radians. However, they all have a negative (-) value as well. This is because there are two solutions for each arc-cosine. The absolute values are the same but the signs are opposite. Essentially, one sign represents the morning and the other sign represents the evening. They are all given positive signs here to avoid confusion.

This next illustration is a summary of the preceding declared and defined basic variable in the order of precedence.

The Basics of Solar Energy and Solar Power

Given: latitude = The Terrestrial latitude in degrees with respect to the Terrestrial Equator. N = (+) and S = (-).
Given: declination = The Celestial declination in degrees with respect to the Celestial Equator. N = (+) and S = (-).
Given: obstruction = The obstruction angle in degrees with respect to the level horizon. Above = (+) and Below = (-).
Given: slope = The positive angle of the slope in degrees with respect to the level horizon.
Given: direction = The compass azimuth that the slope faces in degrees with respect to the Terrestrial North Pole. N = 0/360 deg, E = 90 deg, S = 180 deg, and W = 270 deg..
Given: sc = The solar constant in kilowatts per square meter

$$lat = latitude \cdot \frac{\pi}{180} \qquad b = \frac{\pi}{2} - lat \qquad dec = declination \cdot \frac{\pi}{180}$$

$$obs = obstruction \cdot \frac{\pi}{180} \qquad ao = \frac{\pi}{2} - obs \qquad c = \frac{\pi}{2} - dec$$

$$ax = slope \cdot \frac{\pi}{180} \qquad \gamma x = direction \cdot \frac{\pi}{180}$$

$$p = \cos(b) \cdot \cos(c) \qquad q = \sin(b) \cdot \sin(c) \qquad \alpha o = \operatorname{acos}\left(\frac{\cos(ao) - p}{q}\right)$$

$$bx = \operatorname{acos}(\cos(ax) \cdot \cos(b) + \sin(ax) \cdot \sin(b) \cdot \cos(\gamma x)) \quad \text{(Displaced Lat)}$$

$$\alpha x = \operatorname{acos}\left(\frac{\cos(ax) - \cos(b) \cdot \cos(bx)}{\sin(b) \cdot \sin(bx)}\right) \quad \text{(Displaced Lon)}$$

$$px = \cos(bx) \cdot \cos(c) \qquad qx = \sin(bx) \cdot \sin(c) \qquad \alpha y = \operatorname{acos}\left(\frac{0 - px}{qx}\right)$$

Now we come to the confusing part. This has been saved for the last. There are eight possible configurations for all of the power and energy situations. There is a ninth for the level obstructed plain of the site as well.

Let us first declare and define the variables to be employed. Then we will look at each of the nine cases interdependently.

1. [pw]: This variable represents the reference solar power for an obstructed level plain at the site. It is only applicable if the Sun is shining directly onto the obstructed level plain. This is expressed as kilowatts per square meter.
2. [en]: This variable represents the reference accumulated solar energy for the level obstructed plain of the site. This is expressed as kw-hrs per square meter.
3. [enα]: This variable represents the reference accumulated solar energy as a function of the time of the time of the day with respect to the zenith of the Sun. Before the sunrise with respect to the eastern obstructions it is assigned a value of [0]. After sunset with respect to the western obstruction it is assigned a value equal to the total accumulated solar energy for the day [en]. This is expressed as kw-hrs per square meter.

Here are the reference calculations for a level howbeit obstructed plain at the site.

Case 0: From sunrise on obstructed level plain of site to sunset on obstructed level plain of site.

$$pw = sc \cdot (p + q \cdot \cos(\alpha)) \cdot (-\alpha o \le \alpha \le \alpha o)$$

$$en = sc \cdot \frac{24}{\pi} \cdot (p \cdot \alpha o + q \cdot \sin(\alpha o))$$

$$en\alpha = 0 \quad (\alpha \le -\alpha o) \qquad\qquad en\alpha = en \quad (\alpha > \alpha o)$$

$$en\alpha = sc \cdot \frac{12}{\pi} \cdot (p \cdot \alpha + q \cdot \sin(\alpha)) + \frac{en}{2} \qquad (-\alpha o < \alpha \le \alpha o)$$

Next are the additional variables and calculations for the real situation on the slope located at the site.

1. [θ]: This variable represents a convenient composite for the applicable time of "sunrise." This is expressed as a time adjustment with respect to the apparent zenith of the Sun on the slope. It is expressed as radians,
2. [Φ]: This variable represents a convenient composite for the time of "sunset." This is expressed as a time adjustment with respect to the apparent zenith of the Sun on the slope. It is expressed as radians,
3. [x]: This variable represents the applicable sunrise energy constant for the slope. This is expressed as kw-hrs per square meter.
4. [y]: This variable represents the applicable sunset energy constant for slope. This is expressed as kw-hrs per square meter.
5. [pwx]: This variable represents the real accumulated solar energy for the real day of the slope between the applicable sunrise and the applicable sunset. This is expressed as kilowatts per square meter.
6. [enx]: This variable represents the real accumulated solar energy on the slope for the particular day. This is expressed as kw-hrs per square meter.
7. [enxα]: This variable represents the applicable accumulated solar energy for the slope. If the Sun is not yet shining down on the slope in the morning then it is assigned a value of [0]. If the Sun has later ceased to shine down on the slope then it is assigned a value equal to the total accumulation of solar energy for the day as [enx]. This is expressed as kw-hrs per square meter.

Case 1: Sunrise on site before sunrise on slope
and sunset on site before sunset on slope
for easterly azimuths of slope.

FOR $\gamma \le \pi$ AND $(-1 \cdot \alpha o) \le [-1 \cdot (\alpha y + \alpha x)]$ AND $\alpha o \le (\alpha y - \alpha x)$

$\theta = \alpha y$ $\qquad$ $x = sc \cdot \frac{12}{\pi} \cdot (px \cdot \theta + qx \cdot \sin(\theta))$

$\phi = \alpha x + \alpha o$ $\qquad$ $y = sc \cdot \frac{12}{\pi} \cdot (px \cdot \phi + qx \cdot \sin(\phi))$

$pwx = sc \cdot (px + qx \cdot \cos(\alpha + \alpha x)) \cdot [[-1 \cdot (\alpha y + \alpha x)] \le \alpha \le \alpha o]$

$enx = x + y$

$en x\alpha = 0 \quad [\alpha \le [-1 \cdot (\alpha y + \alpha x)]]$ $\qquad$ $en x\alpha = enx \quad (\alpha > \alpha o)$

$en x\alpha = sc \cdot \frac{12}{\pi} \cdot [px \cdot (\alpha + \alpha x) + qx \cdot \sin(\alpha + \alpha x)] + x \quad [[-1 \cdot (\alpha y + \alpha x)] < \alpha \le \alpha o]$

Case 2: Sunrise on site before sunrise on slope
and sunset on site after sunset on slope
for easterly azimuths of slope.

FOR $\gamma \le \pi$ AND $(-1 \cdot \alpha o) \le [-1 \cdot (\alpha y + \alpha x)]$ AND $\alpha o > \alpha y - \alpha x$

$\theta = \alpha y$ $\qquad$ $x = sc \cdot \frac{12}{\pi} \cdot (px \cdot \theta + qx \cdot \sin(\theta))$

$\phi = \alpha y$ $\qquad$ $y = sc \cdot \frac{12}{\pi} \cdot (px \cdot \phi + qx \cdot \sin(\phi))$

$pwx = sc \cdot (px + qx \cdot \cos(\alpha + \alpha x)) \cdot [[-1 \cdot (\alpha y + \alpha x)] \le \alpha \le (\alpha y - \alpha x)]$

$enx = x + y$

$en x\alpha = 0 \quad [\alpha \le [-1 \cdot (\alpha y + \alpha x)]]$ $\qquad$ $en x\alpha = enx \quad [\alpha > (\alpha y - \alpha x)]$

$en x\alpha = sc \cdot \frac{12}{\pi} \cdot [px \cdot (\alpha + \alpha x) + qx \cdot \sin(\alpha + \alpha x)] + x \quad [-1 \cdot (\alpha y + \alpha x)] < \alpha \le (\alpha y - \alpha x)$

Case 3: Sunrise on site after sunrise on slope
and Sunset on site before sunset on slope
for easterly azimuths of slope.

FOR $\gamma \le \pi$ AND $(-1 \cdot \alpha o) > [-1 \cdot (\alpha y + \alpha x)]$ AND $\alpha o \le (\alpha y - \alpha x)$

$\theta = \alpha o - \alpha x \qquad x = sc \cdot \frac{12}{\pi} \cdot (px \cdot \theta + qx \cdot \sin(\theta))$

$\phi = \alpha o + \alpha x \qquad y = sc \cdot \frac{12}{\pi} \cdot (px \cdot \phi + qx \cdot \sin(\phi))$

$pwx = sc \cdot (px + qx \cdot \cos(\alpha + \alpha x)) \cdot (-\alpha o \le \alpha \le \alpha o)$

$enx = x + y$

$enx\alpha = 0 \quad (\alpha \le -\alpha o) \qquad enx\alpha = enx \quad (\alpha > \alpha o)$

$enx\alpha = sc \cdot \frac{12}{\pi} \cdot [px \cdot (\alpha + \alpha x) + qx \cdot \sin(\alpha + \alpha x)] + x \qquad (-\alpha o < \alpha \le \alpha o)$

Case 4: Sunrise on site after sunrise on slope
and Sunset on site after sunset on slope
for easterly azimuths of slope.

FOR $\gamma \le \pi$ AND $(-1 \cdot \alpha o) > [-1 \cdot (\alpha y + \alpha x)]$ AND $\alpha o > \alpha y - \alpha x$

$\theta = \alpha o - \alpha x \qquad x = sc \cdot \frac{12}{\pi} \cdot (px \cdot \theta + qx \cdot \sin(\theta))$

$\phi = \alpha y \qquad y = sc \cdot \frac{12}{\pi} \cdot (px \cdot \phi + qx \cdot \sin(\phi))$

$pwx = sc \cdot (px + qx \cdot \cos(\alpha + \alpha x)) \cdot [-\alpha o \le \alpha \le (\alpha y - \alpha x)]$

$enx = x + y$

$enx\alpha = 0 \quad (\alpha \le -\alpha o) \qquad enx\alpha = enx \quad [\alpha > (\alpha y - \alpha x)]$

$enx\alpha = sc \cdot \frac{12}{\pi} \cdot [px \cdot (\alpha + \alpha x) + qx \cdot \sin(\alpha + \alpha x)] + x \qquad [-\alpha o < \alpha \le (\alpha y - \alpha x)]$

Case 5: Sunrise on site before sunrise on slope
and sunset on site before sunset on slope
for westerly azimuths of slope.

FOR $\gamma > \pi$ AND $(-1 \cdot \alpha o) \le [-1 \cdot (\alpha y - \alpha x)]$ AND $\alpha o \le (\alpha y + \alpha x)$

$\theta = \alpha y$ $\qquad x = sc \cdot \frac{12}{\pi} \cdot (px \cdot \theta + qx \cdot \sin(\theta))$

$\phi = \alpha o - \alpha x$ $\qquad y = sc \cdot \frac{12}{\pi} \cdot (px \cdot \phi + qx \cdot \sin(\phi))$

$pwx = sc \cdot (px + qx \cdot \cos(\alpha - \alpha x)) \cdot [[-1 \cdot (\alpha y - \alpha x)] \le \alpha \le \alpha o]$

$enx = x + y$

$enx\alpha = 0 \quad [\alpha \le [-1 \cdot (\alpha y - \alpha x)]]$ $\qquad enx\alpha = enx \quad (\alpha > \alpha o)$

$enx\alpha = sc \cdot \frac{12}{\pi} \cdot [px \cdot (\alpha - \alpha x) + qx \cdot \sin(\alpha - \alpha x)] + x \quad [[-1 \cdot (\alpha y - \alpha x)] < \alpha \le \alpha o]$

Case 6: Sunrise on site before sunrise on slope
and sunset on site after sunset on slope
for westerly azimuths of slope.

FOR $\gamma > \pi$ AND $(-1 \cdot \alpha o) \le [-1 \cdot (\alpha y - \alpha x)]$ AND $\alpha o > \alpha y + \alpha x$

$\theta = \alpha y$ $\qquad x = sc \cdot \frac{12}{\pi} \cdot (px \cdot \theta + qx \cdot \sin(\theta))$

$\phi = \alpha y$ $\qquad y = sc \cdot \frac{12}{\pi} \cdot (px \cdot \phi + qx \cdot \sin(\phi))$

$pwx = sc \cdot (px + qx \cdot \cos(\alpha - \alpha x)) \cdot [[-1 \cdot (\alpha y - \alpha x)] \le \alpha \le (\alpha y + \alpha x)]$

$enx = x + y$

$enx\alpha = 0 \quad [\alpha \le [-1 \cdot (\alpha y - \alpha x)]]$ $\qquad enx\alpha = enx \quad [\alpha > (\alpha y + \alpha x)]$

$enx\alpha = sc \cdot \frac{12}{\pi} \cdot [px \cdot (\alpha - \alpha x) + qx \cdot \sin(\alpha - \alpha x)] + x \quad [-1 \cdot (\alpha y - \alpha x)] < \alpha \le (\alpha y + \alpha x)$

Case 7: Sunrise on site after sunrise on slope
and Sunset on site before sunset on slope
for westerly azimuths of slope.

FOR $\gamma > \pi$ AND $(-1\cdot\alpha o) > [-1\cdot(\alpha y - \alpha x)]$ AND $\alpha o \le (\alpha y + \alpha x)$

$\theta = \alpha o + \alpha x \qquad x = sc\cdot\frac{12}{\pi}\cdot(px\cdot\theta + qx\cdot\sin(\theta))$

$\phi = \alpha o - \alpha x \qquad y = sc\cdot\frac{12}{\pi}\cdot(px\cdot\phi + qx\cdot\sin(\phi))$

$pwx = sc\cdot(px + qx\cdot\cos(\alpha - \alpha x))\cdot[(-1\cdot\alpha o) \le \alpha \le \alpha o]$

$enx = x + y$

$enx\alpha = 0 \quad [\alpha \le (-1\cdot\alpha o)] \qquad enx\alpha = enx \quad (\alpha > \alpha o)$

$enx\alpha = sc\cdot\frac{12}{\pi}\cdot[px\cdot(\alpha - \alpha x) + qx\cdot\sin(\alpha - \alpha x)] + x \quad (-1\cdot\alpha o) < \alpha \le \alpha o$

Case 8: Sunrise on site after sunrise on slope
and Sunset on site after sunset on slope
for westerly azimuths of slope.

FOR $\gamma \le \pi$ AND $(-1\cdot\alpha o) > [-1\cdot(\alpha y - \alpha x)]$ AND $\alpha o > (\alpha y + \alpha x)$

$\theta = \alpha o + \alpha x \qquad x = sc\cdot\frac{12}{\pi}\cdot(px\cdot\theta + qx\cdot\sin(\theta))$

$\phi = \alpha y \qquad y = sc\cdot\frac{12}{\pi}\cdot(px\cdot\phi + qx\cdot\sin(\phi))$

$pwx = sc\cdot(px + qx\cdot\cos(\alpha - \alpha x))\cdot[-\alpha o \le \alpha \le (\alpha y + \alpha x)]$

$enx = x + y$

$enx\alpha = 0 \quad (\alpha \le -\alpha o) \qquad enx\alpha = enx \quad [\alpha > (\alpha y + \alpha x)]$

$enx\alpha = sc\cdot\frac{12}{\pi}\cdot[px\cdot(\alpha - \alpha x) + qx\cdot\sin(\alpha - \alpha x)] + x \quad [-\alpha o < \alpha \le (\alpha y + \alpha x)]$

Local Meteorology

In the real world the precise calculations for solar power and solar energy are fine as a beginning, but must be tempered with the realities of clouds, rain, snow, ice, and haze. It is recommended that a meteorological journal be kept of the site. For the purposes of solar energy and solar power the following items should be logged in the journal.

1. The power of the solar radiation per square meter for the level surface of the site in as short of intervals as reasonable.
2. The power of the solar radiation per square meter of the level surface of the slope of interest in as short of intervals as reasonable.
3. The air temperature of the site.
4. The ground temperature of the site.
5. The ground temperature of the slope of interest.
6. Measure of snow, ice, and leaves.

The accumulated energy for each sampling interval may be approximated by multiplying each power record by the length of the interval with respect to the length of an hour. This calculation is then summed up through each day. This too should be logged in the journal as the following.

1. Accumulated energy for the level plain in the same intervals as the power.
2. Accumulated energy for the slope of interest in the same intervals as the power on the slope of interest.
3. Accumulated energy for the day for the level plain of the site.
4. Accumulated energy for the day for the slope of interest.

All this should be recorded in an ongoing journal for a minimum of one year. A minimum of three years would be preferable. This will allow for local probability approximations.