Calculating for the Position of the Sun and the Received Solar Power and Energy for a Given Surface on the Earth (or any Other World)

The First Observatory

A long long time ago on a world far far away, (in time that is), a hunter taking a break set his spear upright in a shallow hole in the Earth. He had been out hunting since the break of dawn and it was now early morning with the Sun just above the rim of the far horizon.

The ground was level and the early morning Sun cast a shadow of the shaft and the spearhead on the Earth below. The hunter watched the shadow of the point of the spearhead move over the ground. Out of idle curiosity having nothing better to do at the time, the hunter began placing small stones at various locations touched by the shadow of the spearhead. This was the first observatory.

Towards evening as the setting Sun was low above the distant horizon the hunter returned to his village. During the day he had bagged some mighty fine quails and a couple of measly hares, and NO, on the way home, he did NOT meet up with a great big grizzly bear!!!

The next day in front of his hut the hunter set a pointed shaft into the Earth with the point up and the butt down. Next to it he set a collection of small stones. His curiosity had been aroused. As the Sun moved across the sky he would now and then position one of the stones where the point of the shaft cast its shadow on the Earth. His wife and the other villagers thought he was nuts. However, since he was a mighty hunter they let him enjoy his new eccentricity. His children were delighted.

The hunter continued on in his chosen profession. He brought down many animals, more than enough to take care of his family, with a surplus to trade with the other villagers for their surpluses. He was doing well.

When he was not hunting or cleaning and tanning his kills he continued to track the shadow of the Sun and record on pieces of bark the results of his observations.

Over a years time he observed that on most days the shadow of the Sun followed a curved path. During the spring and summer it would curve towards the North. During the fall and winter it would curve to the South. He observed that twice a year the shadow of the Sun would follow a straight path from west to east on the days of the spring and fall equinoxes respectively.

What had begun as an idle curiosity during a break from his hunting had now developed into an obsession. However, the hunter did not let his obsession interfere with his necessary employment.

Out of curiosity, the hunter now took a woven framework of brushwood matted with large leaves as to make an opaque surface and inserted a short wooden shaft perpendicular to it. He then began to prop up this arrangement so that the shaft faced North. He now observed that a certain angle the shadow of the Sun moved in a perfect circle. He observed that during the spring and summer the shaft needed to be on top and that during the fall and winter the shaft needed to be below,(where it faced South). This was the first working heliochronometer (precision sundial). He observed that the radius of the circle of the shadow of the Sun was the shortest at the solstices and expanded to infinity at the equinoxes where it would “flip over.”

The hunter began to observe that the Moon, the Stars, and the “wanderers” (planets) in the sky seemed to follow the same patterns as the Sun. He became capable of predicting Celestial events and was as much admired for his astrological knowledge as for his hunting prowess at which he excelled. The villagers no longer considered him to be an eccentric nutcase, his wife was proud of him, and his children desired to follow in his footsteps.

In time the hunter became too old to hunt and settled down to his retirement in the village as the village’s first high priest.

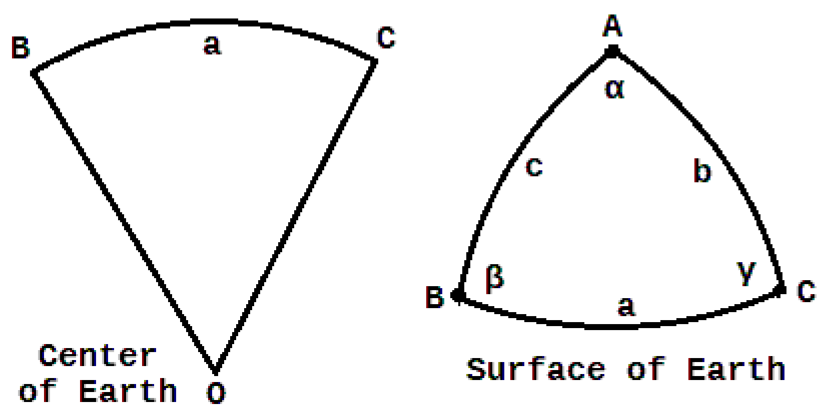
Solar Calculations

Spherical Trigonometry

This study relies heavily on spherical trigonometry. More precisely it relies heavily on the law of cosines from spherical trigonometry.

Given three points on the surface of the Earth; If a man stands at any one point, the other two points will lie on a straight line over the curvature of the Earth from him. If the point that the man is standing on is envisioned as a pole, then the two straight lines radiating out from the point may likewise be envisioned as meridians. In spherical trigonometry the distances between the point that the man is standing on and the two distant points are expressed as angles with respect to the center of the Earth. In addition, there will also be a horizontal angle of separation between the two “meridians.” It does not matter which of the points that the man is standing on.

In this study, the points are indicated in uppercase English, the “sides” are indicated in lowercase English, and the separation angles are indicated in lowercase Greek.



$$\cos(a) = \cos(b)\cdot\cos(c) + \sin(b)\cdot\sin(c)\cdot\cos(\alpha)$$

$$\cos(b) = \cos(a)\cdot\cos(c) + \sin(a)\cdot\sin(c)\cdot\cos(\beta)$$

$$\cos(c) = \cos(a)\cdot\cos(b) + \sin(a)\cdot\sin(b)\cdot\cos(\gamma)$$

“As above so below, as below so above. In Heaven as on Earth, on Earth as in Heaven.” These are fundamental principles underlying many philosophies and religions. These are also the foundations of navigational, geographical, and astrological calculations. Spherical Trigonometry is the mathematical tool that is used to work out these calculations.

The Celestial Sphere has an apparent rotation about the Terrestrial Sphere. If the Celestial Sphere is “frozen” at a point defined by an agreed on convention, Then points of interest on the Terrestrial Sphere may be projected onto the Celestial Sphere and points of interest on the Celestial Sphere may be projected onto the Terrestrial Sphere.

When employing spherical trigonometry, the “sides” of a trilateral wrapped upon the surface of the sphere are expressed as angles with respect to the center of the sphere. Likewise, the more obvious “angles” between the adjacent “sides” are expressed as angles as well.

The preceding illustration depicts a graphic representation of spherical trigonometry as well as the law of cosines with respect to spherical trigonometry. Using cosines is preferable because the cosines and the arc-cosines permit the full range of angles without any acrobatics. The three formulas given are the basic forms. Other forms may be obtained by manipulating these three basic forms.

The use of natural radians as an expression of angles is preferred. Other angles by cultural conventions may be used but conversions will be required. Here are the two most essential conversion factors for the subject at hand.

$$\frac{\text{degree}}{\text{radian}} = \frac{180}{\pi} = 57.296 \qquad \frac{\text{hour}}{\text{radian}} = \frac{12}{\pi} = 3.82$$

Calculating the Altitude of the Sun as a Function of the Time of Day

For these arguments the following will apply:

1. [α]: This variable will represent the time of the day with respect to the zenith of the Sun (High Noon).
2. [a]: This variable will represent the altitude of the Sun with respect to a point directly above the observer.
3. [b]: This variable will represent the Terrestrial latitude of the observer with respect to the Terrestrial North Pole.
4. [c]: This variable will represent the Celestial declination of the Sun with respect to the Celestial North Pole.
5. [alt]: This variable will represent the altitude of the Sun with respect to the level horizon of the observer.
6. [lat]: This variable will represent the Terrestrial latitude of the observer with respect to the Terrestrial Equator [N = (+)].
7. [dec]: This variable will represent the Celestial declination of the Sun with respect to the Celestial Equator [N = (+)].

$$\cos(a) = \cos(b)\cdot\cos(c) + \sin(b)\cdot\sin(c)\cdot\cos(\alpha)$$

$$\cos(b) = \cos\left(\frac{\pi}{2} - lat\right) = \sin(lat)$$

$$-\pi \le \alpha \le \pi$$

$$\sin(b) = \sin\left(\frac{\pi}{2} - lat\right) = \cos(lat)$$

$$\cos(c) = \cos\left(\frac{\pi}{2} - dec\right) = \sin(dec)$$

$$\sin(c) = \sin\left(\frac{\pi}{2} - dec\right) = \cos(dec)$$

$$\cos(a) = \cos\left(\frac{\pi}{2} - alt\right) = \sin(alt)$$

$$a = \operatorname{acos}(\cos(a))$$

$$alt = \frac{\pi}{2} - a$$

$$\sin(alt) = \sin(lat)\cdot\sin(dec) + \cos(lat)\cdot\cos(dec)\cdot\cos(\alpha)$$

Calculating the Time of the Day as a Function of the Altitude of the Sun

For these arguments the following will apply:

1. [α]: This variable will represent the time of the day with respect to the zenith of the Sun (High Noon).
2. [a]: This variable will represent the altitude of the Sun with respect to a point directly above the observer.
3. [b]: This variable will represent the Terrestrial latitude of the observer with respect to the Terrestrial North Pole.
4. [c]: This variable will represent the Celestial declination of the Sun with respect to the Celestial North Pole.
5. [alt]: This variable will represent the altitude of the Sun with respect to the level horizon of the observer.
6. [lat]: This variable will represent the Terrestrial latitude of the observer with respect to the Terrestrial Equator [N = (+)].
7. [dec]: This variable will represent the Celestial declination of the Sun with respect to the Celestial Equator [N = (+)].

$$a = \frac{\pi}{2} - alt \qquad b = \frac{\pi}{2} - lat \qquad c = \frac{\pi}{2} - dec$$

$$\cos(\alpha) = \frac{\cos(a) - \cos(b)\cdot\cos(c)}{\sin(b)\cdot\sin(c)} \qquad \boxed{-\pi \le \alpha \le \pi}$$

$$\cos(\alpha) = \frac{\sin(alt) - \sin(lat)\cdot\sin(dec)}{\cos(lat)\cdot\cos(dec)} \qquad \boxed{\text{Alternate}}$$

$$\alpha = (+)\ \mathrm{acos}(\cos(\alpha)) \quad \text{AND} \quad \alpha = (-)\ \mathrm{acos}(\cos(\alpha))$$

$$\text{IF } a = \frac{\pi}{2} \text{ THEN } \alpha = +/-\ \mathrm{acos}\left(\frac{\cos(b)\cdot\cos(c)}{\sin(b)\cdot\sin(c)}\right)$$

$$\text{IF } alt = 0 \text{ THEN } \alpha = +/-\ \mathrm{acos}\left(\frac{\sin(lat)\cdot\sin(dec)}{\cos(lat)\cdot\cos(dec)}\right)$$

Time of Natural Sunrise and Sunset

Calculating the Compass Azimuth of the Sun as a Function of the Altitude of the Sun

For these arguments the following will apply:

1. [γ]: This variable will represent the plus or minus azimuth of the Sun with respect to “Due North” of the compass.
2. [a]: This variable will represent the altitude of the Sun with respect to a point directly above the observer.
3. [b]: This variable will represent the Terrestrial latitude of the observer with respect to the Terrestrial North Pole.
4. [c]: This variable will represent the Celestial declination of the Sun with respect to the Celestial North Pole.
5. [alt]: This variable will represent the altitude of the Sun with respect to the level horizon of the observer.
6. [lat]: This variable will represent the Terrestrial latitude of the observer with respect to the Terrestrial Equator [N = (+)].
7. [dec]: This variable will represent the Celestial declination of the Sun with respect to the Celestial Equator [N = (+)].

$$a = \frac{\pi}{2} - alt \qquad b = \frac{\pi}{2} - lat \qquad c = \frac{\pi}{2} - dec$$

$$\cos(\gamma) = \frac{\cos(c) - \cos(a)\cdot\cos(b)}{\sin(a)\cdot\sin(b)} \qquad \boxed{-\pi \le \gamma \le \pi}$$

$$\cos(\alpha) = \frac{\sin(dec) - \sin(alt)\cdot\sin(lat)}{\cos(alt)\cdot\cos(lat)} \qquad \boxed{\text{Alternate}}$$

$$\gamma = (+)\ \operatorname{acos}(\cos(\gamma)) \quad \text{AND} \quad \gamma = (-)\ \operatorname{acos}(\cos(\gamma))$$

$$\text{IF } a = \frac{\pi}{2} \text{ THEN } \gamma = +/-\ \operatorname{acos}\left(\frac{\cos(a)\cdot\cos(b)}{\sin(a)\cdot\sin(b)}\right)$$

$$\text{IF } alt = 0 \text{ THEN } \gamma = +/-\ \operatorname{acos}\left(\frac{\sin(alt)\cdot\sin(lat)}{\cos(alt)\cdot\cos(lat)}\right)$$

Azimuth of Natural Sunrise and Sunset

[nadir ≤ altitude ≤ zenith]

Calculating the Altitude of the Sun as a Function of the Compass Azimuth of the Sun

For these arguments the following will apply:

1. [y]: This variable will represent the plus or minus azimuth of the Sun with respect to “Due North” of the compass.
2. [a]: This variable will represent the altitude of the Sun with respect to a point directly above the observer.
3. [b]: This variable will represent the Terrestrial latitude of the observer with respect to the Terrestrial North Pole.
4. [c]: This variable will represent the Celestial declination of the Sun with respect to the Celestial North Pole.
5. [alt]: This variable will represent the altitude of the Sun with respect to the level horizon of the observer.
6. [lat]: This variable will represent the Terrestrial latitude of the observer with respect to the Terrestrial Equator [N = (+)].
7. [dec]: This variable will represent the Celestial declination of the Sun with respect to the Celestial Equator [N = (+)].

The first three of these problems have been fairly straightforward. This next one is not so accommodating, This will involve employing the infamous, horror of horrors, quadratic equation. The problem is that the basic formula that needs to be modified has both a cos(a) and a sin(a). It never advisable to use a sine or an arc-sine if possible. It is far better to use a cosine or an arc-cosine. In order to obtain a cos(a) from a sin(a), the sin(a) must be squared and subtracted from one. This will result in the square of cos(a). However, in the process, there is a cos(a) that is not squared. This is the reason for using the quadratic equation.

The following is the worked quadratic. It was a difficult because there were two non-simultaneous solutions, both of which were required for the full scope.

$$\cos(c) = \cos(a)\cdot\cos(b) + \sin(a)\cdot\sin(b)\cdot\cos(\gamma)$$

$$a = \frac{\pi}{2} - alt$$

$$b = \frac{\pi}{2} - lat$$

$$c = \frac{\pi}{2} - dec$$

$$\cos(c) - \cos(a)\cdot\cos(b) = \sin(a)\cdot\sin(b)\cdot\cos(\gamma)$$

$$\cos(c)^2 - 2\cdot\cos(a)\cdot\cos(b)\cdot\cos(c) + \cos(a)^2\cdot\cos(b)^2 = \sin(a)^2\cdot\sin(b)^2\cdot\cos(\gamma)^2$$

$$\sin(a)^2 = 1 - \cos(a)^2$$

$$\cos(c)^2 - 2\cdot\cos(a)\cdot\cos(b)\cdot\cos(c) + \cos(a)^2\cdot\cos(b)^2 = \left(1 - \cos(a)^2\right)\cdot\sin(b)^2\cdot\cos(\gamma)^2$$

$$\cos(c)^2 - 2\cdot\cos(a)\cdot\cos(b)\cdot\cos(c) + \cos(a)^2\cdot\cos(b)^2 = \sin(b)^2\cdot\cos(\gamma)^2 - \cos(a)^2\cdot\sin(b)^2\cdot\cos(\gamma)^2$$

$$\begin{aligned} 0 = {} & \cos(a)^2\cdot\cos(b)^2 + \cos(a)^2\cdot\sin(b)^2\cdot\cos(\gamma)^2 \\ & - 2\cdot\cos(a)\cdot\cos(b)\cdot\cos(c) \\ & + \cos(c)^2 - \sin(b)^2\cdot\cos(\gamma)^2 \end{aligned}$$

$$\begin{aligned} 0 = {} & \cos(a)^2\cdot\left(\cos(b)^2 + \sin(b)^2\cdot\cos(\gamma)^2\right) \\ & - \cos(a)\cdot(2\cdot\cos(b)\cdot\cos(c)) \\ & + \left(\cos(c)^2 - \sin(b)^2\cdot\cos(\gamma)^2\right) \end{aligned}$$

$$x = \cos(b)^2 + \sin(b)^2\cdot\cos(\gamma)^2$$

$$y = 2\cdot\cos(b)\cdot\cos(c)$$

$$z = \cos(c)^2 - \sin(b)^2\cdot\cos(\gamma)^2$$

$$\cos(a) = \frac{y - \sqrt{y^2 - 4\cdot x\cdot z}}{2\cdot x} \qquad \gamma < \frac{\pi}{2}$$

$$\cos(a) = \frac{y + \sqrt{y^2 - 4\cdot x\cdot z}}{2\cdot x} \qquad \gamma > \frac{\pi}{2}$$

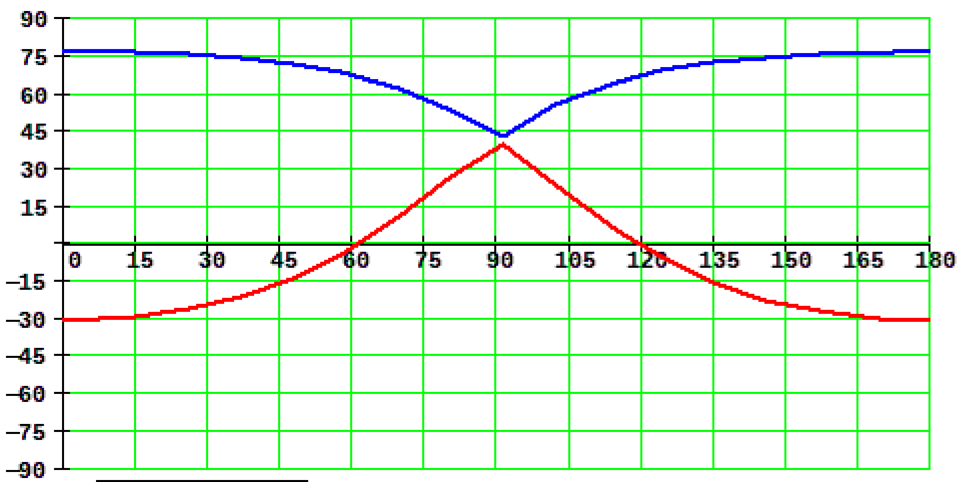
$$a = \operatorname{acos}\left(\frac{y - \sqrt{y^2 - 4\cdot x\cdot z}}{2\cdot x}\right) \quad 0 < \gamma < \frac{\pi}{2} \qquad a = \operatorname{acos}\left(\frac{y + \sqrt{y^2 - 4\cdot x\cdot z}}{2\cdot x}\right) \quad \frac{\pi}{2} < \gamma < \pi$$

Here is a graph of the preceding for the summer solstice at 36° North Latitude. This illustration clearly shows the two solutions and their appropriate applications.

$b := (90-36)\cdot deg \qquad c := (90-23)\cdot deg \qquad y := 2\cdot\cos(b)\cdot\cos(c)$

$$90-\frac{\operatorname{acos}\left[\dfrac{y-\sqrt{y^2-4\cdot\left(\cos(b)^2+\sin(b)^2\cdot\cos(\gamma)^2\right)\cdot\left(\cos(c)^2-\sin(b)^2\cdot\cos(\gamma)^2\right)}}{2\cdot\left(\cos(b)^2+\sin(b)^2\cdot\cos(\gamma)^2\right)}\right]}{deg}$$

$$90-\frac{\operatorname{acos}\left[\dfrac{y+\sqrt{y^2-4\cdot\left(\cos(b)^2+\sin(b)^2\cdot\cos(\gamma)^2\right)\cdot\left(\cos(c)^2-\sin(b)^2\cdot\cos(\gamma)^2\right)}}{2\cdot\left(\cos(b)^2+\sin(b)^2\cdot\cos(\gamma)^2\right)}\right]}{deg}$$



$deg = \dfrac{\pi}{180}$ $\qquad \dfrac{\gamma}{deg}$

This solution will be needed again for insertion of [a] in [α(γ)] where [a] is used as an intermediary. Here is the basic quadratic form. Both solutions always apply in their place.

Quadratic Equation For the form $u^2\cdot x+u\cdot y+z=0$

$$u = \frac{-y-\sqrt{y^2-4\cdot x\cdot z}}{2\cdot x} \quad \textbf{AND} \quad u = \frac{-y+\sqrt{y^2-4\cdot x\cdot z}}{2\cdot x}$$

Calculating the Compass Azimuth of the Sun as a Function of the Time of the Day

For these arguments the following will apply:

1. [α]: This variable will represent the plus or minus time of the day with respect to the zenith of the Sun (High Noon).
2. [γ]: This variable will represent the plus or minus azimuth of the Sun with respect to "Due North" of the compass.
3. [a]: This variable will represent the altitude of the Sun with respect to a point directly above the observer. It shall be used here as an intermediary insertion.
4. [b]: This variable will represent the Terrestrial latitude of the observer with respect to the Terrestrial North Pole.
5. [c]: This variable will represent the Celestial declination of the Sun with respect to the Celestial North Pole.
6. [alt]: This variable will represent the altitude of the Sun with respect to the level horizon of the observer.
7. [lat]: This variable will represent the Terrestrial latitude of the observer with respect to the Terrestrial Equator [N = (+)].
8. [dec]: This variable will represent the Celestial declination of the Sun with respect to the Celestial Equator [N = (+)].

$$-\pi \le \alpha \le \pi \qquad b = \frac{\pi}{2} - lat \qquad c = \frac{\pi}{2} - dec$$

$$a = acos\left(cos(b) \cdot cos(c) + sin(b) \cdot sin(c) \cdot cos(\alpha)\right)$$

$$\gamma = acos\left(\frac{cos(c) - cos(a) \cdot cos(b)}{sin(a) \cdot sin(b)}\right)$$

In this formula, [b] an [c] are both given constants. [α] represents the rotation of the Celestial Sphere (Sun). [a(α)] is calculated first to use as as an intermediary for calculating [γ(α)].

Calculating the Time of the Day as a Function of the Compass Azimuth of the Sun

For these arguments the following will apply:

1. [α]: This variable will represent the plus or minus time of the day with respect to the zenith of the Sun (High Noon).
2. [γ]: This variable will represent the plus or minus azimuth of the Sun with respect to "Due North" of the compass.
3. [a]: This variable will represent the altitude of the Sun with respect to a point directly above the observer. It shall be used here as an intermediary insertion.
4. [b]: This variable will represent the Terrestrial latitude of the observer with respect to the Terrestrial North Pole.
5. [c]: This variable will represent the Celestial declination of the Sun with respect to the Celestial North Pole.
6. [alt]: This variable will represent the altitude of the Sun with respect to the level horizon of the observer.
7. [lat]: This variable will represent the Terrestrial latitude of the observer with respect to the Terrestrial Equator [N = (+)].
8. [dec]: This variable will represent the Celestial declination of the Sun with respect to the Celestial Equator [N = (+)].

$$b = \frac{\pi}{2} - lat \qquad c = \frac{\pi}{2} - dec \qquad 0 \le \gamma \le \pi$$

$$x = \cos(b)^2 + \sin(b)^2 \cdot \cos(\gamma)^2 \qquad \cos(a) = \frac{y - \sqrt{y^2 - 4 \cdot x \cdot z}}{2 \cdot x} \qquad 0 \le \gamma \le \frac{\pi}{2}$$

$$y = 2 \cdot \cos(b) \cdot \cos(c)$$

$$z = \cos(c)^2 - \sin(b)^2 \cdot \cos(\gamma)^2 \qquad \cos(a) = \frac{y + \sqrt{y^2 - 4 \cdot x \cdot z}}{2 \cdot x} \qquad \frac{\pi}{2} < \gamma \le \pi$$

$$\alpha = +/- \operatorname{acos}\left(\frac{\cos(a) - \cos(b) \cdot \cos(c)}{\sin(b) \cdot \sin(c)}\right)$$

There is no need to take the arc-cosine of [cos(a)]. [cos(a)] can simply be dropped as-is into the slot for [cos(a] in the second equation. There is no [sin(a] to contend with.

The Received Power of the Sun for a Level Surface as a Function of the Time of the Day

For these arguments the following will apply:

1. [α]: This variable will represent the plus or minus time of the day with respect to the zenith of the Sun (High Noon).
2. [a]: This variable will represent the altitude of the Sun with respect to a point directly above the observer.
3. [b]: This variable will represent the Terrestrial latitude of the observer with respect to the Terrestrial North Pole.
4. [c]: This variable will represent the Celestial declination of the Sun with respect to the Celestial North Pole.
5. [alt]: This variable will represent the altitude of the Sun with respect to the level horizon of the observer.
6. [lat]: This variable will represent the Terrestrial latitude of the observer with respect to the Terrestrial Equator [N = (+)].
7. [dec]: This variable will represent the Celestial declination of the Sun with respect to the Celestial Equator [N = (+)].
8. [pow]: This variable will represent the amount of power received by the Sun for a given area at right angles to the Sun.
9. [sol]: This variable will represent the seasonal solar constant.

$$a = \frac{\pi}{2} - alt \quad b = \frac{\pi}{2} - lat \quad c = \frac{\pi}{2} - dec \quad -\pi \le \alpha \le \pi$$

$$\cos(a) = \cos(b)\cdot\cos(c) + \sin(b)\cdot\sin(c)\cdot\cos(\alpha)$$

$$pow = sol\cdot\cos(a) = sol\cdot\cos\left(\frac{\pi}{2} - alt\right) = sol\cdot\sin(alt) \quad a < \frac{\pi}{2}$$

$$pow = sol\cdot(\cos(b)\cdot\cos(c) + \sin(b)\cdot\sin(c)\cdot\cos(\alpha)) \quad alt > 0$$

The convention at this time is to express the both the power and the solar constant as kilowatts per square meter. The mean solar constant for the Earth is 1.373 kilowatts per square meter.

The Accumulated Energy of the Sun for a Level Surface as a Function of the Time of the Day

For these arguments the following will apply:

1. [α]: This variable will represent the plus or minus time of the day with respect to the zenith of the Sun (High Noon).
2. [a]: This variable will represent the altitude of the Sun with respect to a point directly above the observer.
3. [b]: This variable will represent the Terrestrial latitude of the observer with respect to the Terrestrial North Pole.
4. [c]: This variable will represent the Celestial declination of the Sun with respect to the Celestial North Pole.
5. [alt]: This variable will represent the altitude of the Sun with respect to the level horizon of the observer.
6. [lat]: This variable will represent the Terrestrial latitude of the observer with respect to the Terrestrial Equator [N = (+)].
7. [dec]: This variable will represent the Celestial declination of the Sun with respect to the Celestial Equator [N = (+)].
8. [ene]: This variable will represent the accumulation of energy received by the Earth for a given area at right angles to the Sun.
9. [sol]: This variable will represent the seasonal solar constant.

In addition to these nine variables there are three more variables of interest.

1. [p]: This variable is for convenience. [p] represents the constant product of [cos(b)] and [cos(c)].
2. [q]: This variable is for convenience. [q] represents the constant product of [sin(b)] and [sin(c)].
3. [d]: This variable represents a convenient holding cell for the time of sunrise and sunset for an unobstructed level horizon.

$$a = \frac{\pi}{2} - alt \qquad b = \frac{\pi}{2} - lat \qquad c = \frac{\pi}{2} - dec$$

$$p = \cos(b)\cdot\cos(c) \qquad q = \sin(b)\cdot\sin(c)$$

$$power = sol\cdot(p + q\cdot\cos(\alpha))$$

$$\int [sol\cdot(p + q\cdot\cos(\alpha))]\, d\alpha = sol\cdot(p\cdot\alpha + q\cdot\sin(\alpha))$$

$$\begin{matrix}\cos(\alpha)\\ \text{Sunset}\\ \text{Sunrise}\end{matrix} = \frac{\cos\left(\frac{\pi}{2}\right) - p}{q} = \frac{-p}{q} \qquad d = acos\left(\frac{-p}{d}\right) \begin{matrix}\text{Time of}\\ \text{Sunrise}\\ \text{Sunset}\end{matrix}$$

$$ene = [sol\cdot(p\cdot\alpha + q\cdot\sin(\alpha)) + sol\cdot(p\cdot d + q\cdot\sin(d))]\cdot\frac{12}{\pi}$$

$$\begin{matrix}\text{Total Accumulated Energy}\\ \text{From Sunrise to Sunset}\end{matrix} = sol\cdot(p\cdot d + q\cdot\sin(d))\cdot\frac{24}{\pi}$$

These equations were all processed in terms of radians. Consequently, There are three issues to be addressed.

The time of sunrise and sunset is “simplified” for the special case of a level unobstructed plain. That situation does not represent the real world. There are always obstructions jutting up from the level horizon.

$$\text{IF } a = \frac{\pi}{2} - obstruction \qquad \text{THEN} \qquad d = acos\left(\frac{\cos(a) - p}{q}\right)$$

The formula for the accumulated energy is taken as an integral equation to the formula for the power. The issue here is that the simple returns will express the accumulated energy as kilowatt-radians. However, our conventions require kilowatt-hours. Thus, we multiply the simple returns by [12/π].

Likewise, for the total accumulated energy, we multiply the simple returns by [24/π]. The last part of the formula is a constant that needs to be multiplied by a factor of [2].

Trigonometric Tables

The following is a useful table of the trigonometric functions. DEG represents the angle in degree. HRA represents the angle in hours. RAD represents the angle in radians. SIN represents the sine of the angle. COS represents the cosine of the angle. TAN represents the tangent of the angle.

UsinU and UcosU are special cases. They represent the cases where an equation has been reduced to a Usin(U) or a Ucos(U) form. In both cases the value of the left-hand [U] MUST be expressed as radians. The orbital time equation is a case in point.

DEG	HRA	RAD	SIN	COS	TAN	UsinU	UcosU
0	0.000	0.000	0.000	1.000	0.000	0.000	0.000
1	0.067	0.017	0.017	1.000	0.017	0.000	0.017
2	0.133	0.035	0.035	0.999	0.035	0.001	0.035
3	0.200	0.052	0.052	0.999	0.052	0.003	0.052
4	0.267	0.070	0.070	0.998	0.070	0.005	0.070
5	0.333	0.087	0.087	0.996	0.087	0.008	0.087
6	0.400	0.105	0.105	0.995	0.105	0.011	0.104
7	0.467	0.122	0.122	0.993	0.123	0.015	0.121
8	0.533	0.140	0.139	0.990	0.141	0.019	0.138
9	0.600	0.157	0.156	0.988	0.158	0.025	0.155
10	0.667	0.175	0.174	0.985	0.176	0.030	0.172
11	0.733	0.192	0.191	0.982	0.194	0.037	0.188
12	0.800	0.209	0.208	0.978	0.213	0.044	0.205
13	0.867	0.227	0.225	0.974	0.231	0.051	0.221
14	0.933	0.244	0.242	0.970	0.249	0.059	0.237
15	1.000	0.262	0.259	0.966	0.268	0.068	0.253
16	1.067	0.279	0.276	0.961	0.287	0.077	0.268
17	1.133	0.297	0.292	0.956	0.306	0.087	0.284
18	1.200	0.314	0.309	0.951	0.325	0.097	0.299
19	1.267	0.332	0.326	0.946	0.344	0.108	0.314
20	1.333	0.349	0.342	0.940	0.364	0.119	0.328
21	1.400	0.367	0.358	0.934	0.384	0.131	0.342
22	1.467	0.384	0.375	0.927	0.404	0.144	0.356
23	1.533	0.401	0.391	0.921	0.424	0.157	0.370
24	1.600	0.419	0.407	0.914	0.445	0.170	0.383
25	1.667	0.436	0.423	0.906	0.466	0.184	0.395
26	1.733	0.454	0.438	0.899	0.488	0.199	0.408
27	1.800	0.471	0.454	0.891	0.510	0.214	0.420
28	1.867	0.489	0.469	0.883	0.532	0.229	0.431
29	1.933	0.506	0.485	0.875	0.554	0.245	0.443
30	2.000	0.524	0.500	0.866	0.577	0.262	0.453

31	2.067	0.541	0.515	0.857	0.601	0.279	0.464
32	2.133	0.559	0.530	0.848	0.625	0.296	0.474
33	2.200	0.576	0.545	0.839	0.649	0.314	0.483
34	2.267	0.593	0.559	0.829	0.675	0.332	0.492
35	2.333	0.611	0.574	0.819	0.700	0.350	0.500
36	2.400	0.628	0.588	0.809	0.727	0.369	0.508
37	2.467	0.646	0.602	0.799	0.754	0.389	0.516
38	2.533	0.663	0.616	0.788	0.781	0.408	0.523
39	2.600	0.681	0.629	0.777	0.810	0.428	0.529
40	2.667	0.698	0.643	0.766	0.839	0.449	0.535
41	2.733	0.716	0.656	0.755	0.869	0.469	0.540
42	2.800	0.733	0.669	0.743	0.900	0.490	0.545
43	2.867	0.750	0.682	0.731	0.933	0.512	0.549
44	2.933	0.768	0.695	0.719	0.966	0.533	0.552
45	3.000	0.785	0.707	0.707	1.000	0.555	0.555
46	3.067	0.803	0.719	0.695	1.036	0.578	0.558
47	3.133	0.820	0.731	0.682	1.072	0.600	0.559
48	3.200	0.838	0.743	0.669	1.111	0.623	0.561
49	3.267	0.855	0.755	0.656	1.150	0.645	0.561
50	3.333	0.873	0.766	0.643	1.192	0.668	0.561
51	3.400	0.890	0.777	0.629	1.235	0.692	0.560
52	3.467	0.908	0.788	0.616	1.280	0.715	0.559
53	3.533	0.925	0.799	0.602	1.327	0.739	0.557
54	3.600	0.942	0.809	0.588	1.376	0.762	0.554
55	3.667	0.960	0.819	0.574	1.428	0.786	0.551
56	3.733	0.977	0.829	0.559	1.483	0.810	0.547
57	3.800	0.995	0.839	0.545	1.540	0.834	0.542
58	3.867	1.012	0.848	0.530	1.600	0.858	0.536
59	3.933	1.030	0.857	0.515	1.664	0.883	0.530
60	4.000	1.047	0.866	0.500	1.732	0.907	0.524
61	4.067	1.065	0.875	0.485	1.804	0.931	0.516
62	4.133	1.082	0.883	0.469	1.881	0.955	0.508
63	4.200	1.100	0.891	0.454	1.963	0.980	0.499
64	4.267	1.117	0.899	0.438	2.050	1.004	0.490
65	4.333	1.134	0.906	0.423	2.145	1.028	0.479
66	4.400	1.152	0.914	0.407	2.246	1.052	0.469
67	4.467	1.169	0.921	0.391	2.356	1.076	0.457
68	4.533	1.187	0.927	0.375	2.475	1.100	0.445
69	4.600	1.204	0.934	0.358	2.605	1.124	0.432
70	4.667	1.222	0.940	0.342	2.747	1.148	0.418
71	4.733	1.239	0.946	0.326	2.904	1.172	0.403
72	4.800	1.257	0.951	0.309	3.078	1.195	0.388
73	4.867	1.274	0.956	0.292	3.271	1.218	0.373
74	4.933	1.292	0.961	0.276	3.487	1.242	0.356
75	5.000	1.309	0.966	0.259	3.732	1.264	0.339
76	5.067	1.326	0.970	0.242	4.011	1.287	0.321
77	5.133	1.344	0.974	0.225	4.331	1.309	0.302
78	5.200	1.361	0.978	0.208	4.705	1.332	0.283
79	5.267	1.379	0.982	0.191	5.145	1.353	0.263
80	5.333	1.396	0.985	0.174	5.671	1.375	0.242

81	5.400	1.414	0.988	0.156	6.314	1.396	0.221
82	5.467	1.431	0.990	0.139	7.115	1.417	0.199
83	5.533	1.449	0.993	0.122	8.144	1.438	0.177
84	5.600	1.466	0.995	0.105	9.514	1.458	0.153
85	5.667	1.484	0.996	0.087	11.430	1.478	0.129
86	5.733	1.501	0.998	0.070	14.300	1.497	0.105
87	5.800	1.518	0.999	0.052	19.081	1.516	0.079
88	5.867	1.536	0.999	0.035	28.635	1.535	0.054
89	5.933	1.553	1.000	0.017	57.286	1.553	0.027
90	6.000	1.571	1.000	0.000	UND	1.571	0.000
91	6.067	1.588	1.000	-0.017	-57.294	1.588	-0.028
92	6.133	1.606	0.999	-0.035	-28.637	1.605	-0.056
93	6.200	1.623	0.999	-0.052	-19.082	1.621	-0.085
94	6.267	1.641	0.998	-0.070	-14.301	1.637	-0.114
95	6.333	1.658	0.996	-0.087	-11.430	1.652	-0.145
96	6.400	1.676	0.995	-0.105	-9.514	1.666	-0.175
97	6.467	1.693	0.993	-0.122	-8.144	1.680	-0.206
98	6.533	1.710	0.990	-0.139	-7.115	1.694	-0.238
99	6.600	1.728	0.988	-0.156	-6.314	1.707	-0.270
100	6.667	1.745	0.985	-0.174	-5.671	1.719	-0.303
101	6.733	1.763	0.982	-0.191	-5.145	1.730	-0.336
102	6.800	1.780	0.978	-0.208	-4.705	1.741	-0.370
103	6.867	1.798	0.974	-0.225	-4.332	1.752	-0.404
104	6.933	1.815	0.970	-0.242	-4.011	1.761	-0.439
105	7.000	1.833	0.966	-0.259	-3.732	1.770	-0.474
106	7.067	1.850	0.961	-0.276	-3.487	1.778	-0.510
107	7.133	1.868	0.956	-0.292	-3.271	1.786	-0.546
108	7.200	1.885	0.951	-0.309	-3.078	1.793	-0.582
109	7.267	1.902	0.946	-0.326	-2.904	1.799	-0.619
110	7.333	1.920	0.940	-0.342	-2.747	1.804	-0.657
111	7.400	1.937	0.934	-0.358	-2.605	1.809	-0.694
112	7.467	1.955	0.927	-0.375	-2.475	1.812	-0.732
113	7.533	1.972	0.921	-0.391	-2.356	1.815	-0.771
114	7.600	1.990	0.914	-0.407	-2.246	1.818	-0.809
115	7.667	2.007	0.906	-0.423	-2.145	1.819	-0.848
116	7.733	2.025	0.899	-0.438	-2.050	1.820	-0.888
117	7.800	2.042	0.891	-0.454	-1.963	1.819	-0.927
118	7.867	2.059	0.883	-0.469	-1.881	1.818	-0.967
119	7.933	2.077	0.875	-0.485	-1.804	1.817	-1.007
120	8.000	2.094	0.866	-0.500	-1.732	1.814	-1.047
121	8.067	2.112	0.857	-0.515	-1.664	1.810	-1.088
122	8.133	2.129	0.848	-0.530	-1.600	1.806	-1.128
123	8.200	2.147	0.839	-0.545	-1.540	1.800	-1.169
124	8.267	2.164	0.829	-0.559	-1.483	1.794	-1.210
125	8.333	2.182	0.819	-0.574	-1.428	1.787	-1.251
126	8.400	2.199	0.809	-0.588	-1.376	1.779	-1.293
127	8.467	2.217	0.799	-0.602	-1.327	1.770	-1.334
128	8.533	2.234	0.788	-0.616	-1.280	1.760	-1.375
129	8.600	2.251	0.777	-0.629	-1.235	1.750	-1.417
130	8.667	2.269	0.766	-0.643	-1.192	1.738	-1.458

131	8.733	2.286	0.755	-0.656	-1.150	1.726	-1.500
132	8.800	2.304	0.743	-0.669	-1.111	1.712	-1.542
133	8.867	2.321	0.731	-0.682	-1.072	1.698	-1.583
134	8.933	2.339	0.719	-0.695	-1.036	1.682	-1.625
135	9.000	2.356	0.707	-0.707	-1.000	1.666	-1.666
136	9.067	2.374	0.695	-0.719	-0.966	1.649	-1.707
137	9.133	2.391	0.682	-0.731	-0.933	1.631	-1.749
138	9.200	2.409	0.669	-0.743	-0.900	1.612	-1.790
139	9.267	2.426	0.656	-0.755	-0.869	1.592	-1.831
140	9.333	2.443	0.643	-0.766	-0.839	1.571	-1.872
141	9.400	2.461	0.629	-0.777	-0.810	1.549	-1.912
142	9.467	2.478	0.616	-0.788	-0.781	1.526	-1.953
143	9.533	2.496	0.602	-0.799	-0.754	1.502	-1.993
144	9.600	2.513	0.588	-0.809	-0.727	1.477	-2.033
145	9.667	2.531	0.574	-0.819	-0.700	1.452	-2.073
146	9.733	2.548	0.559	-0.829	-0.675	1.425	-2.113
147	9.800	2.566	0.545	-0.839	-0.649	1.397	-2.152
148	9.867	2.583	0.530	-0.848	-0.625	1.369	-2.191
149	9.933	2.601	0.515	-0.857	-0.601	1.339	-2.229
150	10.000	2.618	0.500	-0.866	-0.577	1.309	-2.267
151	10.067	2.635	0.485	-0.875	-0.554	1.278	-2.305
152	10.133	2.653	0.469	-0.883	-0.532	1.245	-2.342
153	10.200	2.670	0.454	-0.891	-0.510	1.212	-2.379
154	10.267	2.688	0.438	-0.899	-0.488	1.178	-2.416
155	10.333	2.705	0.423	-0.906	-0.466	1.143	-2.452
156	10.400	2.723	0.407	-0.914	-0.445	1.107	-2.487
157	10.467	2.740	0.391	-0.921	-0.424	1.071	-2.522
158	10.533	2.758	0.375	-0.927	-0.404	1.033	-2.557
159	10.600	2.775	0.358	-0.934	-0.384	0.995	-2.591
160	10.667	2.793	0.342	-0.940	-0.364	0.955	-2.624
161	10.733	2.810	0.326	-0.946	-0.344	0.915	-2.657
162	10.800	2.827	0.309	-0.951	-0.325	0.874	-2.689
163	10.867	2.845	0.292	-0.956	-0.306	0.832	-2.721
164	10.933	2.862	0.276	-0.961	-0.287	0.789	-2.751
165	11.000	2.880	0.259	-0.966	-0.268	0.745	-2.782
166	11.067	2.897	0.242	-0.970	-0.249	0.701	-2.811
167	11.133	2.915	0.225	-0.974	-0.231	0.656	-2.840
168	11.200	2.932	0.208	-0.978	-0.213	0.610	-2.868
169	11.267	2.950	0.191	-0.982	-0.194	0.563	-2.895
170	11.333	2.967	0.174	-0.985	-0.176	0.515	-2.922
171	11.400	2.985	0.156	-0.988	-0.158	0.467	-2.948
172	11.467	3.002	0.139	-0.990	-0.141	0.418	-2.973
173	11.533	3.019	0.122	-0.993	-0.123	0.368	-2.997
174	11.600	3.037	0.105	-0.995	-0.105	0.317	-3.020
175	11.667	3.054	0.087	-0.996	-0.087	0.266	-3.043
176	11.733	3.072	0.070	-0.998	-0.070	0.214	-3.064
177	11.800	3.089	0.052	-0.999	-0.052	0.162	-3.085
178	11.867	3.107	0.035	-0.999	-0.035	0.108	-3.105
179	11.933	3.124	0.017	-1.000	-0.017	0.055	-3.124
180	12.000	3.142	0.000	-1.000	0.000	0.000	-3.142

181	12.067	3.159	-0.017	-1.000	0.017	-0.055	-3.159
182	12.133	3.176	-0.035	-0.999	0.035	-0.111	-3.175
183	12.200	3.194	-0.052	-0.999	0.052	-0.167	-3.190
184	12.267	3.211	-0.070	-0.998	0.070	-0.224	-3.204
185	12.333	3.229	-0.087	-0.996	0.087	-0.281	-3.217
186	12.400	3.246	-0.105	-0.995	0.105	-0.339	-3.229
187	12.467	3.264	-0.122	-0.993	0.123	-0.398	-3.239
188	12.533	3.281	-0.139	-0.990	0.141	-0.457	-3.249
189	12.600	3.299	-0.156	-0.988	0.158	-0.516	-3.258
190	12.667	3.316	-0.174	-0.985	0.176	-0.576	-3.266
191	12.733	3.334	-0.191	-0.982	0.194	-0.636	-3.272
192	12.800	3.351	-0.208	-0.978	0.213	-0.697	-3.278
193	12.867	3.368	-0.225	-0.974	0.231	-0.758	-3.282
194	12.933	3.386	-0.242	-0.970	0.249	-0.819	-3.285
195	13.000	3.403	-0.259	-0.966	0.268	-0.881	-3.287
196	13.067	3.421	-0.276	-0.961	0.287	-0.943	-3.288
197	13.133	3.438	-0.292	-0.956	0.306	-1.005	-3.288
198	13.200	3.456	-0.309	-0.951	0.325	-1.068	-3.287
199	13.267	3.473	-0.326	-0.946	0.344	-1.131	-3.284
200	13.333	3.491	-0.342	-0.940	0.364	-1.194	-3.280
201	13.400	3.508	-0.358	-0.934	0.384	-1.257	-3.275
202	13.467	3.526	-0.375	-0.927	0.404	-1.321	-3.269
203	13.533	3.543	-0.391	-0.921	0.424	-1.384	-3.261
204	13.600	3.560	-0.407	-0.914	0.445	-1.448	-3.253
205	13.667	3.578	-0.423	-0.906	0.466	-1.512	-3.243
206	13.733	3.595	-0.438	-0.899	0.488	-1.576	-3.232
207	13.800	3.613	-0.454	-0.891	0.510	-1.640	-3.219
208	13.867	3.630	-0.469	-0.883	0.532	-1.704	-3.205
209	13.933	3.648	-0.485	-0.875	0.554	-1.768	-3.190
210	14.000	3.665	-0.500	-0.866	0.577	-1.833	-3.174
211	14.067	3.683	-0.515	-0.857	0.601	-1.897	-3.157
212	14.133	3.700	-0.530	-0.848	0.625	-1.961	-3.138
213	14.200	3.718	-0.545	-0.839	0.649	-2.025	-3.118
214	14.267	3.735	-0.559	-0.829	0.675	-2.089	-3.096
215	14.333	3.752	-0.574	-0.819	0.700	-2.152	-3.074
216	14.400	3.770	-0.588	-0.809	0.727	-2.216	-3.050
217	14.467	3.787	-0.602	-0.799	0.754	-2.279	-3.025
218	14.533	3.805	-0.616	-0.788	0.781	-2.342	-2.998
219	14.600	3.822	-0.629	-0.777	0.810	-2.405	-2.970
220	14.667	3.840	-0.643	-0.766	0.839	-2.468	-2.941
221	14.733	3.857	-0.656	-0.755	0.869	-2.531	-2.911
222	14.800	3.875	-0.669	-0.743	0.900	-2.593	-2.879
223	14.867	3.892	-0.682	-0.731	0.933	-2.654	-2.846
224	14.933	3.910	-0.695	-0.719	0.966	-2.716	-2.812
225	15.000	3.927	-0.707	-0.707	1.000	-2.777	-2.777
226	15.067	3.944	-0.719	-0.695	1.036	-2.837	-2.740
227	15.133	3.962	-0.731	-0.682	1.072	-2.898	-2.702
228	15.200	3.979	-0.743	-0.669	1.111	-2.957	-2.663
229	15.267	3.997	-0.755	-0.656	1.150	-3.016	-2.622
230	15.333	4.014	-0.766	-0.643	1.192	-3.075	-2.580

231	15.400	4.032	-0.777	-0.629	1.235	-3.133	-2.537
232	15.467	4.049	-0.788	-0.616	1.280	-3.191	-2.493
233	15.533	4.067	-0.799	-0.602	1.327	-3.248	-2.447
234	15.600	4.084	-0.809	-0.588	1.376	-3.304	-2.401
235	15.667	4.102	-0.819	-0.574	1.428	-3.360	-2.353
236	15.733	4.119	-0.829	-0.559	1.483	-3.415	-2.303
237	15.800	4.136	-0.839	-0.545	1.540	-3.469	-2.253
238	15.867	4.154	-0.848	-0.530	1.600	-3.523	-2.201
239	15.933	4.171	-0.857	-0.515	1.664	-3.576	-2.148
240	16.000	4.189	-0.866	-0.500	1.732	-3.628	-2.094
241	16.067	4.206	-0.875	-0.485	1.804	-3.679	-2.039
242	16.133	4.224	-0.883	-0.469	1.881	-3.729	-1.983
243	16.200	4.241	-0.891	-0.454	1.963	-3.779	-1.925
244	16.267	4.259	-0.899	-0.438	2.050	-3.828	-1.867
245	16.333	4.276	-0.906	-0.423	2.144	-3.875	-1.807
246	16.400	4.294	-0.914	-0.407	2.246	-3.922	-1.746
247	16.467	4.311	-0.921	-0.391	2.356	-3.968	-1.684
248	16.533	4.328	-0.927	-0.375	2.475	-4.013	-1.621
249	16.600	4.346	-0.934	-0.358	2.605	-4.057	-1.557
250	16.667	4.363	-0.940	-0.342	2.747	-4.100	-1.492
251	16.733	4.381	-0.946	-0.326	2.904	-4.142	-1.426
252	16.800	4.398	-0.951	-0.309	3.078	-4.183	-1.359
253	16.867	4.416	-0.956	-0.292	3.271	-4.223	-1.291
254	16.933	4.433	-0.961	-0.276	3.487	-4.261	-1.222
255	17.000	4.451	-0.966	-0.259	3.732	-4.299	-1.152
256	17.067	4.468	-0.970	-0.242	4.011	-4.335	-1.081
257	17.133	4.485	-0.974	-0.225	4.331	-4.371	-1.009
258	17.200	4.503	-0.978	-0.208	4.705	-4.405	-0.936
259	17.267	4.520	-0.982	-0.191	5.144	-4.437	-0.863
260	17.333	4.538	-0.985	-0.174	5.671	-4.469	-0.788
261	17.400	4.555	-0.988	-0.156	6.314	-4.499	-0.713
262	17.467	4.573	-0.990	-0.139	7.115	-4.528	-0.636
263	17.533	4.590	-0.993	-0.122	8.144	-4.556	-0.559
264	17.600	4.608	-0.995	-0.105	9.514	-4.582	-0.482
265	17.667	4.625	-0.996	-0.087	11.430	-4.608	-0.403
266	17.733	4.643	-0.998	-0.070	14.300	-4.631	-0.324
267	17.800	4.660	-0.999	-0.052	19.080	-4.654	-0.244
268	17.867	4.677	-0.999	-0.035	28.633	-4.675	-0.163
269	17.933	4.695	-1.000	-0.017	57.277	-4.694	-0.082
270	18.000	4.712	-1.000	0.000	UND	-4.712	0.000
271	18.067	4.730	-1.000	0.017	-57.303	-4.729	0.083
272	18.133	4.747	-0.999	0.035	-28.640	-4.744	0.166
273	18.200	4.765	-0.999	0.052	-19.083	-4.758	0.249
274	18.267	4.782	-0.998	0.070	-14.301	-4.771	0.334
275	18.333	4.800	-0.996	0.087	-11.431	-4.781	0.418
276	18.400	4.817	-0.995	0.105	-9.515	-4.791	0.504
277	18.467	4.835	-0.993	0.122	-8.145	-4.799	0.589
278	18.533	4.852	-0.990	0.139	-7.116	-4.805	0.675
279	18.600	4.869	-0.988	0.156	-6.314	-4.810	0.762
280	18.667	4.887	-0.985	0.174	-5.671	-4.813	0.849

281	18.733	4.904	-0.982	0.191	-5.145	-4.814	0.936
282	18.800	4.922	-0.978	0.208	-4.705	-4.814	1.023
283	18.867	4.939	-0.974	0.225	-4.332	-4.813	1.111
284	18.933	4.957	-0.970	0.242	-4.011	-4.809	1.199
285	19.000	4.974	-0.966	0.259	-3.732	-4.805	1.287
286	19.067	4.992	-0.961	0.276	-3.487	-4.798	1.376
287	19.133	5.009	-0.956	0.292	-3.271	-4.790	1.464
288	19.200	5.027	-0.951	0.309	-3.078	-4.781	1.553
289	19.267	5.044	-0.946	0.326	-2.904	-4.769	1.642
290	19.333	5.061	-0.940	0.342	-2.748	-4.756	1.731
291	19.400	5.079	-0.934	0.358	-2.605	-4.742	1.820
292	19.467	5.096	-0.927	0.375	-2.475	-4.725	1.909
293	19.533	5.114	-0.921	0.391	-2.356	-4.707	1.998
294	19.600	5.131	-0.914	0.407	-2.246	-4.688	2.087
295	19.667	5.149	-0.906	0.423	-2.145	-4.666	2.176
296	19.733	5.166	-0.899	0.438	-2.050	-4.643	2.265
297	19.800	5.184	-0.891	0.454	-1.963	-4.619	2.353
298	19.867	5.201	-0.883	0.469	-1.881	-4.592	2.442
299	19.933	5.219	-0.875	0.485	-1.804	-4.564	2.530
300	20.000	5.236	-0.866	0.500	-1.732	-4.535	2.618
301	20.067	5.253	-0.857	0.515	-1.664	-4.503	2.706
302	20.133	5.271	-0.848	0.530	-1.600	-4.470	2.793
303	20.200	5.288	-0.839	0.545	-1.540	-4.435	2.880
304	20.267	5.306	-0.829	0.559	-1.483	-4.399	2.967
305	20.333	5.323	-0.819	0.574	-1.428	-4.361	3.053
306	20.400	5.341	-0.809	0.588	-1.376	-4.321	3.139
307	20.467	5.358	-0.799	0.602	-1.327	-4.279	3.225
308	20.533	5.376	-0.788	0.616	-1.280	-4.236	3.310
309	20.600	5.393	-0.777	0.629	-1.235	-4.191	3.394
310	20.667	5.411	-0.766	0.643	-1.192	-4.145	3.478
311	20.733	5.428	-0.755	0.656	-1.150	-4.097	3.561
312	20.800	5.445	-0.743	0.669	-1.111	-4.047	3.644
313	20.867	5.463	-0.731	0.682	-1.072	-3.995	3.726
314	20.933	5.480	-0.719	0.695	-1.036	-3.942	3.807
315	21.000	5.498	-0.707	0.707	-1.000	-3.888	3.888
316	21.067	5.515	-0.695	0.719	-0.966	-3.831	3.967
317	21.133	5.533	-0.682	0.731	-0.933	-3.773	4.046
318	21.200	5.550	-0.669	0.743	-0.900	-3.714	4.125
319	21.267	5.568	-0.656	0.755	-0.869	-3.653	4.202
320	21.333	5.585	-0.643	0.766	-0.839	-3.590	4.278
321	21.400	5.603	-0.629	0.777	-0.810	-3.526	4.354
322	21.467	5.620	-0.616	0.788	-0.781	-3.460	4.429
323	21.533	5.637	-0.602	0.799	-0.754	-3.393	4.502
324	21.600	5.655	-0.588	0.809	-0.727	-3.324	4.575
325	21.667	5.672	-0.574	0.819	-0.700	-3.254	4.646
326	21.733	5.690	-0.559	0.829	-0.675	-3.182	4.717
327	21.800	5.707	-0.545	0.839	-0.649	-3.108	4.786
328	21.867	5.725	-0.530	0.848	-0.625	-3.034	4.855
329	21.933	5.742	-0.515	0.857	-0.601	-2.957	4.922
330	22.000	5.760	-0.500	0.866	-0.577	-2.880	4.988

331	22.067	5.777	-0.485	0.875	-0.554	-2.801	5.053
332	22.133	5.794	-0.469	0.883	-0.532	-2.720	5.116
333	22.200	5.812	-0.454	0.891	-0.510	-2.639	5.178
334	22.267	5.829	-0.438	0.899	-0.488	-2.555	5.239
335	22.333	5.847	-0.423	0.906	-0.466	-2.471	5.299
336	22.400	5.864	-0.407	0.914	-0.445	-2.385	5.357
337	22.467	5.882	-0.391	0.921	-0.424	-2.298	5.414
338	22.533	5.899	-0.375	0.927	-0.404	-2.210	5.470
339	22.600	5.917	-0.358	0.934	-0.384	-2.120	5.524
340	22.667	5.934	-0.342	0.940	-0.364	-2.030	5.576
341	22.733	5.952	-0.326	0.946	-0.344	-1.938	5.627
342	22.800	5.969	-0.309	0.951	-0.325	-1.845	5.677
343	22.867	5.986	-0.292	0.956	-0.306	-1.750	5.725
344	22.933	6.004	-0.276	0.961	-0.287	-1.655	5.771
345	23.000	6.021	-0.259	0.966	-0.268	-1.558	5.816
346	23.067	6.039	-0.242	0.970	-0.249	-1.461	5.859
347	23.133	6.056	-0.225	0.974	-0.231	-1.362	5.901
348	23.200	6.074	-0.208	0.978	-0.213	-1.263	5.941
349	23.267	6.091	-0.191	0.982	-0.194	-1.162	5.979
350	23.333	6.109	-0.174	0.985	-0.176	-1.061	6.016
351	23.400	6.126	-0.156	0.988	-0.158	-0.958	6.051
352	23.467	6.144	-0.139	0.990	-0.141	-0.855	6.084
353	23.533	6.161	-0.122	0.993	-0.123	-0.751	6.115
354	23.600	6.178	-0.105	0.995	-0.105	-0.646	6.145
355	23.667	6.196	-0.087	0.996	-0.087	-0.540	6.172
356	23.733	6.213	-0.070	0.998	-0.070	-0.433	6.198
357	23.800	6.231	-0.052	0.999	-0.052	-0.326	6.222
358	23.867	6.248	-0.035	0.999	-0.035	-0.218	6.244
359	23.933	6.266	-0.017	1.000	-0.017	-0.109	6.265
360	24.000	6.283	0.000	1.000	0.000	0.000	6.283